When Can Auctions Maximize Post-Auction Welfare?

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Abstract

I study auctions in which firms bid for licenses that reduce their marginal costs in a post-auction downstream market. When there are three or more firms, I show that the Vickrey–Clarke–Groves (VCG) auction maximizes consumer surplus in dominant strategies if and only if it maximizes producer surplus in dominant strategies. With two firms, the effect on consumer surplus is ambiguous. When the VCG auction does not maximize consumer surplus, I show that consumer surplus can be maximized by adding caps, i.e., restricting the number of licenses a bidder can win. This might lower producer surplus.

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1 Introduction

Many multi-unit auctions allocate essential production inputs to firms. Importantly, the firms compete not only in the auction, but also in a downstream market after the auction. For example, many governments auction spectrum licenses to telecommunication companies. The companies require these licenses for the provision of their services, as only the electromagnetic spectrum can transmit mobile-phone calls and data. Winning more spectrum licenses in an auction allows a firm to transmit more data through its cell towers, which reduces the number of required towers and the cost of maintaining a certain level of capacity (Rey and Salant, 2017).

An important theoretical benchmark, but also an auction format used in practice, is the Vickrey–Clarke–Groves (VCG) auction.[1] The VCG auction was designed to be efficient in the auction-theoretic sense of maximizing bidder welfare in dominant strategies. However, if the bidders' values for the auctioned goods are the expected profits in a post-auction downstream market, then efficiency corresponds to maximizing downstream producer surplus. I study the impact of this "efficient" (i.e., industry-profit maximizing) auction on downstream consumer surplus.

I address this issue with a model in which the auction first allocates multiple marginal cost reducing licenses among a fixed number of firms. After the auction, the firms are in Cournot competition. Explicitly modeling the downstream market allows me to determine endogenously the bidders' values and to study the auction's impact on downstream consumers. Other papers on auctions with a downstream market have largely focused on the extensive margin: How does consumer surplus depend on the auction outcome when the license allocation determines the number or set of post-auction active firms?[2] In contrast, I focus on the intensive margin by assuming that the cost functions are such that all bidders are active in equilibrium after any auction outcome.[3] An important element of my model is the cost-reduction technology, which specifies how the licenses reduce marginal costs.

The first main result shows that the VCG auction maximizes both consumer

[1]Spectrum blocks have been sold via the dynamic Combinatorial Clock Auction (CCA). The final round of the CCA is basically a VCG auction (Cramton, 2013; Janssen and Kasberger, 2019).

[2]Dana and Spier (1994) inquire into the optimal number of active firms. Hoppe et al. (2006) study the impact of the number of entry licenses on the market structure. Gebhardt and Wambach (2008) consider the optimal number of active firms when they have privately known fixed costs.

[3]The intensive margin seems relevant in several real-world settings. For example, in most spectrum auctions all bidders are established incumbents. The incumbents already own other spectrum licenses, so they can be expected to remain active even if they do not win any new licenses. Indeed, Verizon did not buy any new spectrum in the US Incentive Auction (600 MHz) in 2017 and has not exited the market to date. Before the award of the British 2.3 and 3.4 GHz spectrum bands, the regulator assessed it as unlikely that the auction outcome would reduce the number of mobile-network operators from four to three (Ofcom, 2017).

and producer surplus in dominant strategies when the cost-reduction technology is linear. Thus, the VCG auction is not only "efficient" in the auction-theoretic sense of maximizing bidder welfare in dominant strategies, but also truly socially efficient in dominant strategies. It may be surprising that the auction outcome is optimal for consumers even though they do not bid. The optimality follows from the consumers being indifferent between the license allocations: The sum of marginal costs determines aggregate output in Cournot competition and, hence, consumer surplus (Bergstrom and Varian, 1985). When the cost-reduction technology is linear, then the sum of marginal costs is the same for all allocations in which all licenses are sold, which implies the consumers' indifference between these allocations. I also prove the converse: A linear cost-reduction is necessary for the VCG auction to maximize consumer and producer surplus in dominant strategies.

Consumers prefer more equal license allocations when the cost-reduction technology is strictly convex. However, the VCG auction may lead to very unequal allocations and may even minimize consumer surplus. In this setting, I show how caps can improve outcomes in favor of consumers; caps limit how many licenses a bidder can win.[4] Under certain conditions, when consumers' downstream demand is sufficiently high, industry profits have a local maximum at the consumer-surplus maximizing allocation. The VCG auction implements this allocation when caps rule out a potential global maximum on the boundary. The result provides a formal justification for pairing an "efficient" format with caps: The auction locally maximizes industry profits, but the caps ensure that this local maximum is "close" to the consumer-optimal allocation.[5] Importantly, caps do not need to be binding to be effective. However, when downstream demand is not high enough, caps may be binding and consumer surplus may increase with the caps' restrictiveness.

In a related two-firm Hotelling model in which both firms are always active, Mayo and Sappington (2016) analyze an auction that allocates a marginal change in marginal costs to the firm with a more substantial increase in profits. They find that it is not always socially efficient to assign the cost reduction to this firm. In contrast, I analyze the effect of selling multiple licenses to several firms in downstream

[4]It is well understood that caps may enhance downstream competition by facilitating entry (Cramton et al., 2011).

[5] Many practical designs pair an otherwise "efficient" format with caps. For example, spectrum auctions usually seek "efficiency" while using caps. The British regulator Ofcom states that it seeks an efficient allocation "because the operator with the highest value for the spectrum will normally be the one most likely to use the frequencies to deliver the services consumers most want" (Ofcom, 2017). Note that this argument considers a single-unit setting in which the "use value" dominates the "foreclosure value" (Mayo and Sappington, 2016). In any case, for the same sale, concerns that "a very asymmetric distribution of spectrum" might weaken post-auction competition have led to the adoption of spectrum caps (Ofcom, 2017).

Cournot competition. Jehiel and Moldovanu (2000) focus on the externalities that arise in the sale of a single good when the seller may sometimes keep the object. Related allocative and identity-dependent externalities also play a role in my model, in particular when the cost-reduction technology is nonlinear. In this case, a dominant strategy exists only when there are two firms.

As mentioned, the majority of studies of auctions with an after-market focus on the extensive margin: The licenses are entry licenses that determine the set of active firms (see also footnote 2). Rey and Salant (2017) analyze the sale of divisible cost reductions to an incumbent and an entrant in post-auction homogeneous Bertrand competition. In the downstream market, only the firm with the post-auction lower marginal costs will be active in equilibrium, making the licenses essentially entry licenses. For consumers, it is optimal to reduce the costs of the (inactive) firm with the second-highest marginal cost, which might be challenging to implement in practice. Janssen and Karamychev (2010) show in a unit-demand setting that auctions do not necessarily select the firms with the lowest marginal cost. This is in contrast to the case in which there is a single monopoly license for sale (Demsetz, 1968; Laffont and Tirole, 1999). Eső et al. (2010) allocate capacity constraints to ex ante symmetric firms in capacity-constrained Cournot competition with complete information with a bidder-welfare maximizing auction. Note that exogenous capacity constraints also resemble entry licenses. Martimort and Pouyet (2020) study two upstream firms bidding in a second-price auction for additional capacity. While their focus is on how initial capacity impacts bidding, my main interest is how "efficient" multi-unit auctions impact downstream consumers.

The next section presents the model. Section 3 analyzes linear cost-reduction technologies. Section 4 analyzes strictly convex cost-reduction technologies and investigates the role of caps. In addition, I show how set-asides, i.e., non-competitively awarded licenses, may increase consumer surplus in an extension in which not all firms are active after all license allocations. Section 5 concludes. Appendix A provides the omitted proofs. Appendix B characterizes the differentiated Bertrand and Cournot markets for which consumers are indifferent between auction outcomes and for which a dominant strategy exists.

2 The Model

In the model, n firms first bid in a VCG auction for multiple licenses. The firms then learn the outcome of the auction and the licenses reduce the winners' marginal production costs. After the auction, the firms compete in downstream Cournot competition. Licenses cannot be resold after the auction.

The cost-reducing licenses are perfectly divisible and are available in supply with a unit measure. The auction determines a feasible license allocation $x \in \mathbb{R}^n_+$, where $x = (x_1, x_2, \dots, x_n)$ and $x_1 + x_2 + \cdots + x_n \leq 1$. A *no-undersell allocation* is feasible and allocates the entire supply, i.e., $x_1 + \cdots + x_n = 1$. Let X denote the set of all feasible allocations. The set of no-undersell allocations is given by $\bar{X}$.

The Market After the Auction. After the auction, $n \geq 2$ firms are in homogeneous Cournot competition; firm i chooses quantity $q_i \geq 0$ to maximize profits.[6] Industry output is denoted by $Q = q_1 + \cdots + q_n$ and determines the market price through the inverse demand function $P(Q)$. There is a $\bar{Q} > 0$ such that $P(Q) > 0$ and $P'(Q) < 0$ for $Q \in [0, \bar{Q})$ and $P(Q) = 0$ for $Q \geq \bar{Q}$. The inverse demand function is twice continuously differentiable and has the decreasing marginal-revenue property, i.e., $P'(Q) + qP''(Q) < 0$ for all Q with $P(Q) > 0$ and all $q \in [0, Q]$.

Production costs $C_i(q_i, x_i)$ are linear in output so that $C_i(q_i, x_i) = c_i(x_i) \cdot q_i$. The marginal costs $c_i(x_i) = \theta_i + \rho(x_i)$ are the sum of the initial marginal cost θ_i and the effect of the twice continuously differentiable *cost-reduction technology* (CRT) ρ, where $\rho(0) = 0$. I assume that all firms have the same cost-reduction technology, which may be the case when firms use similar technologies and subcontractors. All licenses are effective in reducing marginal costs, so $\rho' < 0$. Marginal costs never become negative, i.e., $c_i(1) \geq 0$. Firm i's profits conditional on auction allocation x and production choices $q = (q_1, \dots, q_n)$ are $\pi(q|x) = P(Q) \cdot q_i - c_i(x_i) \cdot q_i$.

Modeling the industry as Cournot competition captures the capacity choice that characterizes many actual markets. The capacity can be the antennas in a network, an airline fleet, or a fishing fleet. After the capacity choice, firms compete in prices. Under certain conditions, the outcome in the capacity-then-price game equals the outcome of the standard Cournot game (Kreps and Scheinkman, 1983; Wu et al., 2012).[7] For analytical simplicity, I abstract away from the pricing stage and consider standard Cournot competition.

The allocation of production inputs often decreases marginal production costs. For example, in the spectrum auction context, former Federal Communications Commission Chief Technologist Jon Peha argues that the total cost of providing a certain level of capacity q is linear in q (as in my model), and that the marginal cost of capacity is decreasing and convex in the amount of spectrum a firm owns (Peha, 2017). While spectrum reduces marginal costs in my model, I maintain flexibility

[6] Appendix B considers heterogeneous Bertrand and Cournot competition.

[7] Factors that influence the outcome equivalence are the rationing rule (Davidson and Deneckere, 1986) and the cost asymmetry (Deneckere and Kovenock, 1996).

over the shape of the cost reduction.[8]

In my model, firms have perfect and complete information. Complete information approximates well-established firms that know each other and have access to similar technologies, consultants, and business cases.[9] Firms have perfect information as the auction outcome is publicly announced after the auction, as in many real-world auctions. The solution concept is subgame perfect Nash equilibrium. As the auction expenditure is sunk, it is convenient to summarize all subgames that follow the auction allocation $x \in X$ to one Cournot continuation game. The decreasing marginal-revenue property implies the existence of a unique (pure) equilibrium for any $x \in X$ (Vives, 1999).

I restrict attention to markets that lead to interior equilibria: All firms produce a positive quantity after any allocation $x \in X$ in equilibrium. All firms being active after any auction outcome models auctions in which only established incumbents bid. As mentioned in the introduction, in many auctions there are no potential entrants and the technology is not "revolutionary" in the sense that a firm that does not win enough licenses has to exit the market. A key consequence is that a firm that wins all licenses does not become a monopolist after the auction. Section 4.2 analyzes the case in which not all firms are active after all auction allocations.

Let $\pi_i(x)$ denote the indirect profit derived from the unique equilibrium in the Cournot continuation game induced by $x \in X$. Firm i's continuation equilibrium production is denoted by $q_i(x)$. Note that $q_i(x) > 0$ as the equilibrium is interior. The indirect industry profit function is $\pi(x) = \pi_1(x) + \cdots + \pi_n(x)$. Consumer surplus is $CS(x) = \int_0^{Q(x)} P(y)\, dy - P(Q(x))Q(x)$ and is strictly increasing in aggregate output $Q(x)$. Social welfare is the sum of industry profits and consumer surplus.

The Auction Before the Market. A Vickrey–Clarke–Groves auction is held before the market interaction. In this auction, bidders bid on shares by submitting bidding functions $B_i : [0,1] \to \mathbb{R}_+$. The auctioneer then implements the allocation x that maximizes the sum of bids, that is, $x \in \arg\max_{\tilde{x} \in X} \sum_{j=1}^{n} B_j(\tilde{x}_j)$. Every bidder i pays the VCG price $\max_{\tilde{x} \in X} \sum_{j \neq i} B_j(\tilde{x}_j) - \sum_{j \neq i} B_j(x_j)$, which is the reported externality imposed on the other bidders.

A bidding function B_i is a *dominant strategy* when it is a best response against all

[8] Another example is multi-unit procurement auctions with learning-by-doing: A firm that wins a substantial share in the current auction has lower marginal costs in future projects. Under this interpretation, the downstream market summarizes all future interactions. In auctions for airport departure slots, more slots lower the cost of serving the airport.

[9] Private information on production costs would lead to informational externalities and signaling opportunities (Goeree, 2003). I focus on the impact of the auction on the market structure and leave informational concerns to future work.

profiles of other bidders' bidding functions (Krishna, 2010). An auction *maximizes producer (consumer) surplus in dominant strategies* when (1) it has a dominant strategy for every bidder, and (2) a profile of dominant strategies leads to an allocation x^* that maximizes producer (consumer) welfare, that is, $x^* \in \arg\max_x \pi(x)$ ($x^* \in \arg\max_x CS(x)$). Surplus is always net of transfers. In auction-theoretic models without an aftermarket, the allocation is "efficient" if it maximizes bidder welfare. In contrast, I distinguish between producer and consumer surplus; the auction is truly (socially) efficient if it maximizes the sum of these.

The VCG auction with bids on shares has been used as *the* VCG auction in practice. For example, some recent spectrum auctions have used the Combinatorial Clock Auction (CCA), a multi-round auction in which the final round is basically a VCG auction (Levin and Skrzypacz, 2016). The motivation for using the VCG auction stems from its attractive theoretical properties: Under certain assumptions that include the absence of allocative externalities, the VCG auction is the only auction that maximizes bidder welfare in dominant strategies (Nisan, 2007).[10] I do not make these assumptions. Instead, I study how the post-auction market translates into the bidders' willingness-to-pay, so that the auction is efficient in dominant strategies. A key step involves characterizing the absence of allocative externalities. When there are externalities, the VCG auction with bids on entire allocations, and not with bids on shares, is the theoretically "true" VCG auction. Nevertheless, I refer to the VCG auction used in practice as *the* VCG auction.

3 Linear Cost-Reduction Technologies

This section investigates the properties of the VCG auction when firms have a linear cost-reduction technology; a CRT is *linear* when $\rho(x) = r \cdot x$, with $-\theta_i \leq r < 0$ for all firms i. The first proposition shows that the attractive theoretical properties of the VCG auction hold when the cost-reduction technology is linear.

Proposition 1. *Let the cost-reduction technology be linear. The VCG auction maximizes both producer and consumer surplus in dominant strategies.*

The auction not only maximizes bidder welfare (producer surplus) in dominant strategies, it also maximizes consumer surplus. This may be surprising, as the auction is designed to maximize bidder welfare and there is no guarantee that it

[10] There is a literature on allocative and identity-dependent externalities (preferences over auction allocations) in standard auction and mechanism design (Das Varma, 2002; Jehiel and Moldovanu, 2006). The focus is on the welfare of the participating agents, and there are no downstream consumers. Bichler et al. (2017) interpret bidding in the 2015 German spectrum auction in light of allocative externalities.

will lead to good outcomes for consumers who do not participate in the auction. Yet the underlying market is such that the auction leads to optimal outcomes for consumers *and* producers. Hence, the auction is truly socially efficient: It maximizes the sum of producer and consumer surplus in dominant strategies.

To prove the proposition, I first consider which allocations maximize consumer surplus. Homogeneous Cournot competition is an aggregative game with aggregate output Q as the game's aggregate. Moreover, the game is such that the aggregate determines consumer surplus and is itself determined by the sum of marginal costs (Bergstrom and Varian, 1985). To see this directly, consider the sum of the n first-order conditions for individual profit maximization:

$$nP(Q) + P'(Q)Q = \sum_{i=1}^{n} c_i(x_i). \tag{1}$$

The left-hand side decreases in Q due to the decreasing marginal-revenue property; the sum of marginal costs thus uniquely determines the aggregate output Q. It follows that no-undersell license allocations that minimize the sum of marginal costs maximize consumer surplus in general. In the case of a linear CRT, the right-hand side of Equation (1), $\sum_{i=1}^{n} c_i(x_i) = r + \sum_{i=1}^{n} \theta_i$, is constant for all no-undersell allocations, implying that the aggregate Q is the same. Hence, consumers are indifferent between all such allocations. Consumer surplus is then maximized, provided that the auction leads to a no-undersell allocation; this will turn out to be the case, as firms have a positive marginal willingness-to-pay.

Next, consider the existence of a dominant strategy. Recall that the VCG auction is designed so that bidding the willingness-to-pay is dominant for all shares in standard private-value settings. What is firm i willing to pay for x_i? Recall that a linear cost-reduction technology implies that the same equilibrium aggregate output Q is produced for all no-undersell license allocations in which firm i wins x_i. In such allocations, firm i has the same marginal cost $c_i(x_i)$ and the same (inclusive) best reply to this Q (Vives, 1999). Thus, its profits are the same; the firm does not care how the remaining licenses are allocated. The willingness-to-pay for x_i is then the gain in profits over not winning any licenses: $\pi_i(x_i, x_{-i}) - \pi_i(0, x'_{-i})$, where $(x_i, x_{-i}), (0, x'_{-i}) \in \bar{X}$ are arbitrary no-undersell allocations. A standard argument shows that bidding this willingness-to-pay is a dominant strategy.

All bidders bidding their willingness-to-pay implements the allocation that maximizes producer surplus, as the auction selects $x \in \arg\max_{\tilde{x} \in X} \sum_i \pi_i(\tilde{x})$; scaling the bids down by the constant $\pi_i(0, x'_{-i})$ does not affect the final allocation. The bidding functions are increasing because firm i's profits are increasing in x_i and decreasing in x_j due to the decreasing marginal-revenue property (Vives, 1999). Therefore, the auction selects a no-undersell allocation. Appendix A shows that only no-undersell allocations maximize industry profits, which completes the proof of Proposition 1.[11]

To provide some understanding of what the auction outcome may look like, the following example characterizes the outcome for linear inverse-demand functions.

Example 1: Linear inverse-demand and linear cost-reduction technologies. Let $P(Q) = a - b \cdot Q$ and $c_i(x_i) = \theta_i + rx_i$. Standard analysis (Belleflamme and Peitz, 2010) leads to equilibrium output

$$q_i(x) = \frac{a - nc_i(x_i) + \sum_{j \neq i} c_j(x_j)}{b(n+1)}.$$

Firm i's profits are $\pi_i(x) = b \cdot q_i(x)^2$. Observe that π_i is convex and increasing in q_i, and that q_i is convex in x (the Hessian of q_i is positive semi-definite). It follows that the composition π_i is convex in x. Industry profits $\pi = \sum_i \pi_i$ are then also convex. Industry profits are maximized by a no-undersell allocation in which one bidder wins the entire supply (Rockafellar, 1970, Theorem 32.2). Routine algebra verifies that π is highest when the firm with the lowest θ_i wins all the licenses. ◀

This example shows that equilibrium outcomes may be highly asymmetric: The firm with the lowest initial marginal cost θ_i wins the entire supply. The outcome nevertheless maximizes consumer surplus because the sole winner does not become a monopolist; the other firms' cost functions are such that they remain active.[12]

Proposition 1 hinges on the fact that the firms do not care about how the remaining licenses are allocated (as long as the total mass of licenses is held constant)[13] so that the willingness-to-pay is well-defined and a dominant strategy exists. This means that they have preferences over shares as defined as follows:

Definition 1. Firm i has *preferences over shares* if $\pi_i(x) = \pi_i(x')$ for all no-undersell allocations x, $x' \in \bar{X}$ with $x_i = x'_i$.

When the firm does not have preferences over shares, it has *preferences over full license allocations.* The majority of auction models assume preferences over shares. However, in upstream markets for cost reductions, it may be more natural to expect that different allocations in which bidder i wins x_i will lead to different

[11] While this result is intuitive, it does not trivially hold. In fact, Seade (1987) contains an example in which decreasing the costs of *all* firms decreases industry profits. In contrast, for any undersell allocation, Lemma 3 in Appendix A shows that decreasing the costs of the firm with the lowest costs increases industry profits.

[12] A multi-unit auction is not necessary in Example 1: Selling the entire supply as a single good via a second-price auction would be simpler and also socially efficient.

[13] The firms prefer the remaining licenses not to be sold as this raises the competitors' costs.

post-auction profits, as the competitors' marginal costs differ. The next lemma shows that a linear cost-reduction technology is not only sufficient for the absence of such allocative and identity-dependent externalities (as in Proposition 1), but that it is also necessary.[14]

Lemma 1. *Let $n \geq 3$. The following statements are equivalent:*

- *Firms have preferences over shares.*
- *Firms have a dominant strategy in the VCG auction.*
- *The cost-reduction technology is linear.*

The proof of Proposition 1 shows that a linear CRT is sufficient for firms having preferences over shares, and that when they have preferences over shares, they have a dominant strategy in the VCG auction. In Appendix A I show that a firm has a dominant strategy only if it has preferences over shares. Intuitively, when profits depend on the entire allocation, the bidding language of the VCG auction (with bids on shares) is too restrictive to admit a dominant strategy: If bidder i has several "values" for winning x_i, then the optimality of reporting one of these values depends on the bids of the other bidders, meaning that there cannot be a dominant strategy. Moreover, I prove that preferences over shares arise only if the cost reduction is linear: If the cost reduction is not linear, the sum of marginal costs depends on the entire allocation. Different license allocations lead to different Q, and hence each firm's inclusive best reply to Q depends on the license allocation.

A consequence of Lemma 1 is that the positive result of Proposition 1 does not extend beyond linear cost-reduction technologies when there are more than three bidders. (The next section considers the two-bidder case.)

Proposition 2. *Let $n \geq 3$. The VCG auction maximizes producer and consumer surplus in dominant strategies only if the cost-reduction technology is linear.*

The results of the current section imply that, when there are at least three bidders, the VCG auction maximizes producer surplus in dominant strategies if and only if it maximizes consumer surplus in dominant strategies. Hence, when the VCG auction is "efficient in dominant strategies," it is truly socially efficient. While the VCG auction is socially efficient in dominant strategies when the CRT is linear, Peha's engineering-economic model suggests that the marginal costs are strictly convex in x_i (Peha, 2017); a strictly convex CRT is decreasingly effective in reducing marginal costs. In the next section, I study strictly convex cost-reduction technologies and the impact of the VCG auction on consumer surplus.

[14] It is also necessary that the costs C_i are linear in q_i.

4 Convex Cost-Reduction Technologies

In this section I analyze how strictly convex cost-reduction technologies influence the bidders' and consumers' preferences, and auction outcomes. I also study how auction-design tools such as caps and set-asides influence outcomes.

In the two-bidder case, the bidders always have preferences over shares because they can internalize the externalities that arise when the other firm lowers its marginal costs. If bidder i uses a strictly increasing bidding function, then the auction rules guarantee that the auction ends with a no-undersell allocation. Expecting a no-undersell allocation, if bidder i wins x_i, then the other bidder wins $1 - x_i$. Hence, bidder i's willingness-to-pay is $\pi_i(x_i, 1 - x_i) - \pi_i(0, 1)$, which is the gain in profit over not winning any licenses. As the willingness-to-pay is well defined for each share, bidders always have a dominant strategy in the VCG auction. The VCG auction then maximizes producer surplus in dominant strategies.

Proposition 3. *Let $n = 2$. The firms have preferences over shares and the VCG auction maximizes producer surplus in dominant strategies.*

In the case of three or more bidders and a strictly convex CRT, Lemma 1 implies that there is no dominant strategy equilibrium in the VCG auction. It is then natural to weaken the solution concept and study other Nash equilibria. However, the VCG auction is known to have multiple equilibria in the independent private-value setting (Blume et al., 2009). It is not this paper's objective to characterize these equilibria under allocative externalities. Instead, I restrict attention to payoff-dominant equilibria. A Nash equilibrium is payoff dominant if there is no Pareto superior equilibrium (Harsanyi and Selten, 1988). The next proposition proves by construction the existence of a payoff-dominant Nash equilibrium.

Proposition 4. *The VCG auction has a payoff-dominant equilibrium that maximizes downstream producer surplus.*

To understand the auction's impact on consumer surplus, I first characterize the consumer-optimal allocation. Recall from Equation (1) that consumer surplus is determined by the sum of marginal costs and is maximized when the sum of marginal costs is minimized. When the CRT is strictly convex, then the sum of marginal costs is convex and symmetric. The sum of marginal costs is minimized by $\tilde{x} = (1/n, \ldots, 1/n)$, and hence consumer surplus is maximized by $\tilde{x}$. On the other hand, consumer surplus is minimized when one firm wins the entire supply.

The following example illustrates that the impact of the VCG auction on consumer surplus is ambiguous in the case of a strictly convex CRT. Propositions 3

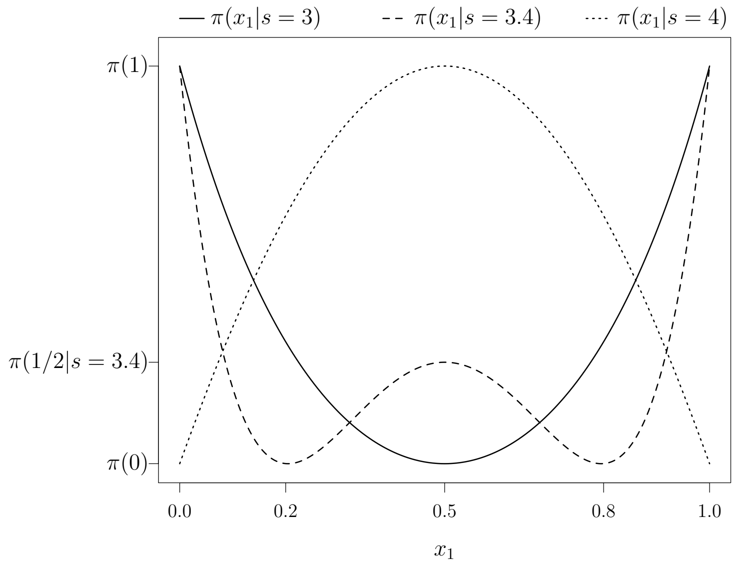

Figure 1: Industry profits as a function of firm 1's share. Figure not to scale.

and 4 suggest that the producer-optimal allocation can be expected as the outcome. The example shows that this allocation may minimize or maximize consumer surplus. Hence, there is not always a trade-off between consumer and producer surplus.

Example 2: The VCG auction and consumer surplus. The inverse demand function is $P(Q) = s(4\log(2) - Q)$, where $s \geq 1$ scales the market size: The larger the s, the larger the consumers' willingness-to-pay in the post-auction market for quantity Q. There are two ex ante identical firms with marginal costs $c(x_i) = \log(2) - \log(1 + x_i)$; the CRT is strictly convex. From the above discussion, it follows that the allocation $(1, 0)$ minimizes consumer surplus on the set of no-undersell allocations, whereas the allocation $\tilde{x}$ maximizes consumer surplus.

Let $i = 1, 2$. As there are two firms and the auction outcome leads to a no-undersell allocation, I write profits π_i as a function of firm i's share. Firm i's equilibrium profits are

$$\pi_i(x_i|s) = \frac{((\log(2)(4s-1) + 2\log(1+x_i) - \log(2-x_i))^2}{9s}.$$

Figure 1 illustrates industry profits as a function of firm 1's share for different values of s. Note that the figure illustrates the shape of the three curves but does not report the true magnitude of the profits. The VCG auction always selects an allocation that maximizes industry profits. It is evident that for $s = 3$, the VCG auction will implement the allocation $(1, 0)$ or $(0, 1)$, which is the worst outcome for consumers. When $s = 4$, the auction outcome maximizes consumer surplus. For the intermediate value $s = 3.4$, industry profits have a local maximum at the consumer-surplus maximizing allocation, but the global maximum of industry profits is on the boundary. The VCG auction's impact on consumer surplus is ambiguous. ◀

4.1 Caps and Consumer Surplus

As the VCG auction may minimize consumer surplus, I now ask whether and how auction-design tools such as caps can improve outcomes for consumers. Caps limit the number of goods a bidder can win and appear in many practical auctions. For example, most spectrum auctions use caps (Ausubel and Baranov, 2017). The effect of caps on producer surplus is straightforward: Caps either rule out the bidder-welfare maximizing allocation (if it is very unequal), or they do not affect the final allocation. The effect of caps on consumer surplus is less clear. Previous studies have often used the number of active firms as a proxy for consumer surplus; limiting the number of licenses the incumbents can win can facilitate or enable entry (Cramton et al., 2011). In the current model, the number of post-auction active firms does not depend on the license allocation.

I study the role of caps in the following markets: Fix an inverse demand function $\tilde{P}$ and consider the inverse demand function $P(Q) = s\tilde{P}(Q)$, where the parameter $s \geq 1$ scales the consumers' demand; a larger s increases the market as measured by equilibrium aggregate output Q. Note that if a unique pure equilibrium exists for the inverse demand function $\tilde{P}$ and profile of cost functions c, then it also exists for inverse demand function $s\tilde{P}$ and cost functions c. For simplicity, suppose that all firms face the same cap $\bar{x}$, where $1/n < \bar{x} < 1$, which specifies that firm i can win at most $\bar{x}$. The next proposition uncovers a novel mechanism of how caps can improve consumer surplus in the standard VCG auction. The caps' effectiveness depends on the consumers' downstream willingness-to-pay.

Proposition 5. *Let the cost-reduction technology be strictly convex, let firms have the same initial marginal costs, and let $P(Q) = s\tilde{P}(Q)$ with $s \geq 1$. For sufficiently large s, there are caps $\bar{x} > 1/n$ such that a payoff-dominant equilibrium of the VCG auction maximizes consumer surplus.*

Figure 1 illustrates the result. Recall that consumer surplus is maximized by the allocation $(1/2, 1/2)$ and minimized on $\bar{X}$ by the allocation $(1, 0)$ in Example 2. Looking at Figure 1, the VCG auction selects the allocation $(1, 0)$ when there is no cap in place and $s = 3$. A cap of $\bar{x}$ restricts the domain to $[\bar{x}, 1 - \bar{x}]$. The maximum of π on the restricted domain is attained at the license allocation $(\bar{x}, 1 - \bar{x})$ with a binding cap. As the auction outcome comes closer to the consumers' bliss point

$\tilde{x}$, consumer surplus decreases in $\bar{x}$. Now consider the high value for s. When demand is high, the cap is ineffective, as there is no trade-off between maximizing producer and consumer surplus. Caps are not binding and do not change the auction outcome. The caps are very effective for intermediate values of s. When $s = 3.4$, industry profits have a local maximum in the interior, but a global maximum on the boundary. For $\bar{x}$ close to 1, the caps are binding and consumer surplus is decreasing in $\bar{x}$. When the caps become sufficiently small, however, they restrict the domain so that the local maximum of industry profits at 1/2 becomes the global maximum. Note that any cap $\bar{x} < \hat{x}$, where $\hat{x}$ is such that $\pi(\hat{x}) = \pi(1/2)$, leads to allocation $\tilde{x}$.

Insights from this example carry over to more general settings. Industry profits have a local maximum at the consumer-surplus maximizing allocation $\tilde{x}$ when the market size s is sufficiently large and the firms are ex ante symmetric. Sufficiently restrictive caps transform this local maximum into the global maximum of the restricted domain. Thus, the result rationalizes real-world auction designs that complement an auction format intended to maximize bidder welfare with caps (see footnote 5). With caps, the auction may maximize producer surplus in a neighborhood of the consumer-surplus maximizing allocation, finding, loosely speaking, a compromise between producer surplus and consumer surplus. Importantly, the caps do not need to be binding to be effective, and there can be a range of optimal caps.

Before discussing asymmetric firms, I explain the intuition behind the result by means of a linear demand function, $P(Q) = s(a - bQ)$. In general, industry profits $P(Q)Q - \sum c_i q_i$ are composed of revenue and total cost. It is ex ante not clear which allocation maximizes industry profits: Minimizing the sum of marginal costs maximizes Q, which might minimize revenue. In the linear case, firm i's equilibrium output is

$$q_i(x) = \frac{as - nc_i(x_i) + \sum_{j \neq i} c_j(x_j)}{(n+1)bs}.$$

Importantly, $q_i(x_i)$ converges to the constant $a/(b(n+1))$ for any license allocation as s tends to infinity. As output does not depend on the license allocation in the limit, the allocation that minimizes the sum of marginal costs maximizes industry profits. This intuition extends to general demand functions.

Proposition 5 applies to ex ante identical firms. Asymmetries in the initial marginal cost do not change the consumer-surplus maximizing allocation but shift the local interior maximum of industry profits. Hence, the VCG auction with caps no longer maximizes consumer surplus, as the following example shows. The caps nevertheless increase consumer surplus.

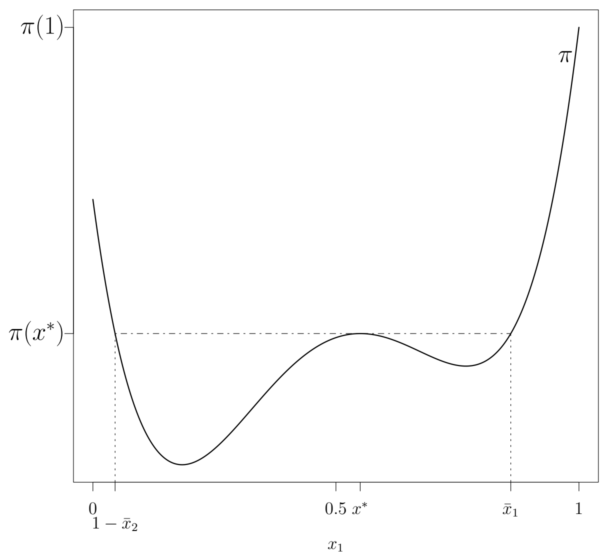

Figure 2: Industry profits with asymmetric initial marginal costs as a function of firm 1's share.

Example 3: Asymmetric marginal costs and the effectiveness of caps. To illustrate the effect of the asymmetries, I slightly modify Example 2. The demand function and firm 1's cost function are as in Example 2. Firm 2, however, has a slightly higher initial marginal cost ($\theta_2 = 0.6941$ instead of $\theta_1 = 0.6931$), so that the industry-profit maximizing allocation is $x_1^* = 0.55$. The consumer-surplus maximizing allocation is still $x_1 = 0.5$. In the case of an intermediate market size ($s = 3.4$), industry profits have a local but not a global maximum in the interior. Figure 2 shows the industry profits as a function of x_1. The global maximum is at $x_1 = 1$; this would be the outcome of an unfettered VCG auction. If there are caps $\bar{x}_1$ and $\bar{x}_2$, then one cap will be binding if both are chosen too low. If both caps are sufficiently restrictive, as are the ones depicted in Figure 2, then the global maximum of industry profits on the restricted domain is interior and at x_1^*. The VCG auction would implement this allocation, which is preferred by consumers over any boundary allocation. However, note that x_1^* is not consumer optimal. ◀

Proposition 5 provides some guidance in choosing caps. First, for a not-too-low downstream demand, there can be a range of optimal caps, as they need only rule out highly asymmetric industry-profit maximizing allocations. Second, the firms' having similar and decreasingly effective cost reductions suggests that caps should equalize the overall license holdings so that the firms are ex post equally strong. Third, caps become less important the larger the downstream market.

4.2 Extension: Entry and Set-Asides

This section analyzes firms' and consumers' preferences when not all firms are active in all Cournot continuation equilibria. It also examines how set-asides may improve auction outcomes in favor of consumer surplus. Set-aside licenses are reserved for a particular bidder or group of bidders. Real-life auctions use set-asides to facilitate entry or to support small businesses (Cramton et al., 2011).[15]

The model is changed as follows: The first $n-1$ firms have cost functions such that they are, as before, active after any auction allocation. However, firm n produces a positive equilibrium quantity only if it wins at least $\epsilon \in (0, 1/n)$. Formally, c_n is prohibitively high for $x_n \in [0, \epsilon)$ and is not necessarily continuous at ϵ. All firms are identical up to firm n's handicap, i.e., $c = c_1 = c_2 = \cdots = c_{n-1}$ and $c_n(x) = c(x)$ for $x \in [\epsilon, 1]$. Firm n may be either an entrant that needs a certain number of licenses to operate its business profitably or a potentially exiting incumbent.

I now discuss the implications for the firms' preferences. First, the handicapped firm's profits are zero for $x_n < \epsilon$ and become positive at ϵ. The firm has preferences over shares if and only if the CRT is linear. The other firms always have preferences over allocations, as it clearly makes a difference how many firms are active in the industry. Moreover, conditional on winning x_i and $x_n < \epsilon$, firm i wants firm n to win as much as possible, as this keeps firm n out of the market and increases the other active firms' costs by reducing the residual license supply.

Consumers prefer more over fewer firms in the market. On the other hand, an active entrant increases the sum of initial marginal costs and raises the other firms' marginal costs. Hence, there are two license allocations that potentially maximize consumer surplus when the CRT is convex. First, $\tilde{x}$ maximizes consumer surplus when all firms are active. Second, $\hat{x} = (1/(n-1), \ldots, 1/(n-1), 0)$ maximizes consumer surplus conditional on firm n not being active. The next proposition shows that $\tilde{x}$ is consumer optimal. However, it cannot be expected that an unfettered VCG auction implements $\tilde{x}$, as industry profits are higher under $\hat{x}$ with $n-1$ active firms.

Proposition 6. *Let the cost-reduction technology be strictly convex and all firms ex ante identical, except that firm n needs to win $\epsilon < 1/n$ to be active in the Cournot continuation equilibrium. The allocation $\tilde{x}$ maximizes consumer surplus. The payoff-dominant equilibrium of the VCG auction does not implement $\tilde{x}$.*

[15] For example, two of six available 800 MHz blocks were reserved for a potential entrant in the 2013 Austrian spectrum auction (RTR, 2013). If there had been more than one qualified bidder, there would have been an auction for the set-aside blocks before the main auction.

Augmenting the auction with set-asides and caps may improve the outcome for consumers. Formally, a set-aside is a share $x^{sa} \in [0,1]$ reserved for firm n via the following procedure: Firm n first decides how many of the set-aside licenses to buy before the auction, i.e., it chooses $y_n \leq x^{sa}$. The remaining licenses $1 - y_n$ are allocated through the regular auction. Firm n can participate in the regular auction after receiving y_n. For simplicity, the price for y_n equals zero.

Proposition 7. *Let the environment be as in Proposition 6 and $P(Q) = s\tilde{P}(Q)$. When s is sufficiently high, then for any set-aside $x^{sa} \in [\epsilon, 1/n]$ there are caps $\bar{x} > 1/n$ so that the payoff-dominant equilibrium of the VCG auction maximizes consumer surplus.*

When the set-aside is at least ϵ, then firm n knows that buying x^{sa} leads to positive post-auction equilibrium profits. Hence, it will buy $y_n = x^{sa}$, as the price of the set-aside licenses is zero. The situation is then as in Proposition 5: When s is sufficiently high, industry profits have a local maximum at $\tilde{x}$. Caps restrict the domain such that the local maximum becomes the global maximum of the restricted domain. The VCG auction then leads to allocation $\tilde{x}$.

Sufficiently tight caps can be a substitute for set-asides. In particular, when $\bar{x} \leq (1-\epsilon)/(n-1)$ then, for a large enough s, the VCG auction with caps and without set-asides maximizes consumer surplus. Conversely, no caps are needed when the market size is sufficiently large that industry profits have a local maximum at $\tilde{x}$ that becomes the global maximum after the set-aside has eliminated the global maximum at $\hat{x}$. Note that some spectrum auctions, such as the Austrian auction in 2013, feature set-asides and caps, which may still be insufficient to attract entrants. Indeed, Marcoux (2022) documents that competition-enhancing policies often fail to attract and sustain entry in the real world.

5 Conclusion

The VCG auction is, under some assumptions, the unique auction that maximizes bidder welfare in dominant strategies (Nisan, 2007). In upstream auctions for marginal cost-reducing goods with at least three firms, the VCG auction (as used in practice) maximizes downstream consumer surplus in dominant strategies if and only if the cost-reduction technology is linear. In the strictly convex case, the impact on consumer surplus is ambiguous and the auction loses the dominant-strategy property.

The paper highlights the important role of caps and set-asides for downstream consumer welfare. Instead of potentially minimizing consumer surplus, the VCG auction complemented with caps and set-asides can lead to the consumer-optimal license allocation. Note, however, that Rey and Salant (2017) document an abandonment of overall spectrum caps in the US. For the same period, Gutiérrez and Philippon (2017) report an increase in concentration of the US telecommunications industry. Other countries have also witnessed a decline in the number of mobile-network operators in the last decade (Rey and Salant, 2017). Too lenient caps and set-asides are a possible explanation for these industry trends.

A Omitted Proofs

Proposition 1 follows from the discussion in the main text and the following Lemma 3, which proves that only no-undersell allocations maximize industry profits. However, I first establish a technical result that will be used in subsequent proofs.

Lemma 2. *Let $i = 2, \dots, n$. The partial derivatives of equilibrium production are*

$$\frac{\partial q_1(x)}{\partial x_1} = \frac{c_1'(x_1)\,(nP'(Q) + Q_{-1}P''(Q))}{P'(Q)\,((n+1)P'(Q) + QP''(Q))} \tag{2}$$

$$\frac{\partial q_i(x)}{\partial x_1} = -\frac{c_1'(x_1)\,(P'(Q) + q_iP''(Q))}{P'(Q)\,((n+1)P'(Q) + QP''(Q))}, \tag{3}$$

respectively. For $i = 1, \dots, n$, the partial derivative of aggregate output is

$$\frac{\partial Q(x)}{\partial x_i} = \frac{c_i'(x_i)}{(n+1)P'(Q) + QP''(Q)}. \tag{4}$$

Proof of Lemma 2. The proof uses the implicit function theorem. Given allocation x, bidder j selects q_j to maximize profits. Profit maximization yields the FOC

$$F^j(x, q_j) = P'(Q_{-j}(x) + q_j)q_j + P(Q_{-j}(x) + q_j) - c_j(x_j) = 0. \tag{5}$$

Let F^j_y be the partial derivative of F^j with respect to y. The implicit function theorem (Simon and Blume, 1994, IFT) implies

$$\frac{\partial q_j(x)}{\partial x_1} = -\frac{F^j_{x_1}}{F^j_{q_j}} = -\frac{\sum_{k\neq j} \frac{\partial q_k}{\partial x_1}(P''q_j + P') - \frac{\partial c_j(x_j)}{\partial x_1}}{P''q_j + 2P'}. \tag{6}$$

This leads to a system of n linear equations in n unknowns. I now verify that equations (2) and (3) solve these. Let $j = 1$. Plugging equations (2) and (3) into the previous yields

$$\frac{c_1'\left(nP'+Q_{-1}P''\right)}{P'\left((n+1)P'+QP''\right)} = -\frac{\left(P''q_1+P'\right)\sum_{i\neq 1} -\frac{c_1'(P'+q_iP'')}{P'((n+1)P'+QP'')} - c_1'}{P''q_1+2P'}.$$

Dividing by c_1', simplifying, and solving the sum transforms the equation to

$$\begin{aligned}
&\left(nP'+Q_{-1}P''\right)\left(P''q_1+2P'\right) = \\
&\quad = P'\left((n+1)P'+QP''\right) + (P''q_1+P')((n-1)P'+Q_{-1}P'') \\
&\quad = (n+1)(P')^2 + P'P''(Q+(n-1)q_1+Q_{-1}) + (n-1)(P')^2 + q_1Q_{-1}(P'')^2 \\
&\quad = 2n(P')^2 + P'P''(nq_1+2Q_{-1}) + q_1Q_{-1}(P'')^2 = \left(nP'+Q_{-1}P''\right)\left(P''q_1+2P'\right).
\end{aligned}$$

Consider Equation (6) with $j=i$ and plug in equations (2) and (3). Multiplying with $P'\left((n+1)P'+QP''\right)$ yields

$$-c_1'\left(P'+q_iP''\right) = -\frac{\left(P''q_i+P'\right)\left(c_1'\left(nP'+Q_{-1}P''\right) - \sum_{k\neq 1,i} c_1'\left(P'+q_kP''\right)\right)}{\left(P''q_i+2P'\right)},$$

which is equivalent to the true statement

$$\left(P''q_i+2P'\right) = \left(nP'+Q_{-1}P'' - (n-2)P' - (Q_{-1}-q_i)P''\right).$$

For industry output, applying the implicit function theorem to the sum of n first-order conditions of profit maximization (Eq. (1)) directly gives Equation (4). □

Lemma 3. *An undersell allocation cannot maximize industry profits.*

Proof of Lemma 3. Suppose that industry profits are maximized by the allocation x with $x \in X \setminus \bar{X}$ and $x \in \arg\max_{\tilde{x}\in X}\pi(\tilde{x})$. Without loss of generality, let bidder 1 be the bidder with the lowest costs, i.e., $c_1(x_1) \leq c_j(x_j)$ for all $j=1,\ldots,n$. I will show that industry profits increase in x_1. Writing industry profits and output as a function of x_1 and taking the derivative yields

$$\begin{aligned}
\pi'(x_1) &= Q'(P'Q+P) - c_1'q_1 - \sum_{i=1}^{n} c_iq_i' \\
&= \frac{c_1'(P'Q+P)}{(n+1)P'+QP''} - c_1'q_1 + \frac{-c_1c_1'\left(nP'+Q_{-1}P''\right) + \sum_{i\neq 1} c_ic_1'\left(P'+q_iP''\right)}{P'\left((n+1)P'+QP''\right)},
\end{aligned}$$

where I use the expressions for Q' (Eq. (4)) and q_i' (Eq. (2) and (3)) in the last step. I simplify to

$$\frac{c_1'(P'(P'Q+P) - q_1 P'\left((n+1)P' + QP''\right) - c_1\left(nP' + Q_{-1}P''\right) + \sum_{i\neq 1} c_i\left(P' + q_i P''\right))}{P'\left((n+1)P' + QP''\right)}.$$

Note that $c_1'/(P'\left((n+1)P' + QP''\right)) < 0$ as $c_1' < 0$, $P' < 0$, and $(n+1)P' + QP'' < 0$ because of decreasing marginal revenues. Hence, $\pi' > 0$ if and only if

$$P'(P'Q+P) - q_1 P'\left((n+1)P' + QP''\right) - c_1\left(nP' + Q_{-1}P''\right) + \sum_{i\neq 1} c_i\left(P' + q_i P''\right) < 0.$$

I will show that this inequality is true by simplifying the left-hand side to

$$P'(P'Q + P - q_1(n+1)P' - nc_1 + \sum_{i\neq 1} c_i) + P''(-q_1 QP' - c_1 Q_{-1} + \sum_{i\neq 1} c_i q_i).$$

I use the sum of FOCs (Eq. (1)) to substitute $P'Q = \sum_{i=1}^{n} c_i - nP$ and the FOC to substitute $q_i P' = c_i - P$ to obtain

$$2P'(P - c_1 + \sum_{i\neq 1} c_i - (n-1)c_1) + P''(Q(P - c_1) + \sum_{i\neq 1}(c_i - c_1)q_i).$$

Further transformations yield

$$\underbrace{2(P - c_1 + \sum_{i\neq 1} c_i - (n-1)c_1)}_{>0}\left(P' + P''\underbrace{\frac{Q(P-c_1) + \sum_{i\neq 1}(c_i - c_1)q_i}{2(P - c_1 + \sum_{i\neq 1} c_i - (n-1)c_1)}}_{\geq 0}\right). \quad (7)$$

The first factor is positive as $P > c_i$ and $c_i \geq c_1$ for all i. The factor that is multiplied with P'' is nonnegative. I will now show that

$$\frac{Q(P-c_1) + \sum_{i\neq 1}(c_i - c_1)q_i}{2(P - c_1 + \sum_{i\neq 1} c_i - (n-1)c_1)} \leq Q. \quad (8)$$

Simplifying leads to the inequality $\sum_{i\neq 1}(c_i - c_1)q_i \leq Q(P - c_1) + 2Q\sum_{i\neq 1}(c_i - c_1)$. The inequality is true as $\sum_{i\neq 1}(c_i - c_1)q_i \leq \sum_{i\neq 1}(c_i - c_1)q_1 \leq 2Q\sum_{i\neq 1}(c_i - c_1)$, where I use that $q_1 \geq q_i$ for all i as firm 1 has the lowest marginal costs.

As a result, I can write the right factor of Equation (7) as $P'(Q) + P''(Q)q$, where $q \in [0, Q]$. By the decreasing marginal-revenue property, this expression is negative. I conclude that $\pi' > 0$. □

Proof of Lemma 1. The main text argues that a linear CRT implies preferences over shares, and preferences over shares imply the existence of a dominant strategy. The proof shows that preferences over shares imply a linear CRT and that a dominant strategy implies preferences over shares.

I first show that if the firms have preferences over shares, then the CRT must be linear. Wlog, suppose bidder 1 wins x_1 in a no-undersell allocation. Bidder 1's continuation profit is $\max_{q_1} P(Q_{-1} + q_1)q_1 - c_1 q_1$. The derivative with respect to Q_{-1} is $P'q_1$ according to the envelope theorem, which is strictly negative. Hence, bidder 1 is indifferent between the no-undersell allocations only if Q_{-1} does not depend on the specific allocation. Given that firm 1's best response to Q_{-1} is the same for all such allocations, so is Q. As the left-hand side of Eq. (1) is independent of the no-undersell allocation in which firm 1 wins x_1, the sum of marginal costs must also be independent of the allocation. It is straightforward to see that the sum of marginal costs is the same for all $x \in \bar{X}$ if and only if the CRT is linear.

I now show that a dominant strategy implies preferences over shares. Let bidder i have a dominant strategy B_i and let $x^* \in \bar{X}$ such that $x_i^* < 1$. The bidders who win nothing in x^* bid 0 for all shares and are henceforth ignored. I now construct bidding functions for the other bidders so that x^* is implemented. Note that there are no restrictions on the bids of the other bidders. Let bidder $j \neq i$ bid 0 on all shares except on a neighborhood around x_j^*. In that neighborhood, B_j is continuously differentiable and strictly increasing with $B_j'(x_j^*) = B_i'(x_i^*)$. The second derivative of B_j at x_j^* is sufficiently negative so that x^* is indeed a local maximum of $\sum_k B_k$ s.t. $\sum_k x_k \leq 1$. The level of $B_j(x_j^*)$ is sufficiently high that there are no boundary solutions. The auctioneer solves the problem $\max_{x \in X} \sum_{k=1}^{n} B_k(x_k)$. The first-order conditions of the corresponding Lagrangian yields $B_j'(x_j^*) = B_k'(x_k^*)$ for those who win positive shares.

Winning x_i^* must be locally optimal for firm i given B_{-i}: If B_i was not a dominant strategy, the firm would alter its marginal bids around x_i^*. Hence, x^* must locally maximize $\pi_i(x) + \sum_{j \neq i} B_j(x_j)$. Plugging in the resource constraint $x_j = 1 - \sum_{k \neq j} x_k$ and taking the derivative with respect to x_i gives the first-order condition $\partial \pi_i(x^*)/\partial x_i = B_j'(x_j^*)$. As the right-hand side is always equal to the constant $B_i(x_i^*)$, the left-hand side $\partial \pi_i(x^*)/\partial x_i$ is the same for all no-undersell allocations in which firm i wins x_i. This means that the profit increases in x_i with the same slope for all allocations in which i's share is constant. Hence, if there were two no-undersell allocations in which firm i wins the same amount but the profits are different, then the same-slope condition implies different profits in the allocation in which firm i wins the entire supply (due to the fundamental theorem of calculus). This clearly cannot be the case; the firm must have preferences over shares. Consider the argument more formally: Let $x, x' \in \bar{X}$ such that $x_i = x'_i$ and $\pi_i(x) < \pi_i(x')$. The same-slope condition implies that there is a function f such that $f(x_i) = \partial\pi_i(y_i, x_{-i})/\partial y_i|_{y_i=x_i}$ for $x \in \bar{X}$. Then $\pi_i(1) = \pi_i(x) + \int_{x_i}^1 f(t)dt$ and $\pi_i(1) = \pi_i(x') + \int_{x_i}^1 f(t)dt$, where $\pi_i(1)$ is short notation for firm i's profit when it wins the entire supply. □

Proof of Proposition 4. Let x denote the feasible allocation that maximizes the welfare of the bidders, i.e., $x \in \arg\max_{\tilde{x}\in Y} \sum_{i=1}^n \pi_i(\tilde{x})$, where Y is the set of feasible allocations. Let n^* be the number of winners in allocation x, i.e., the number of bidders for which $x_i > 0$. Let π^* denote the highest profit any bidder can derive from any allocation, i.e., $\pi^* = \max_i \max_{\tilde{x}\in X} \pi_i(\tilde{x})$.

The following bidding strategies form a Nash equilibrium and lead to the allocation x: Bidders who receive a positive share bid π^* on x_i and 0 on all other shares. Other bidders bid 0 on all quantities.

The profile of these bidding functions clearly implements the allocation x. The only other allocations that receive positive bids are those in which not all shares are sold. These allocations do not maximize the sum of bids, however.

I now show that no player has an incentive to deviate. The VCG price for a winner is $(n^* - 1)\pi^* - (n^* - 1)\pi^* = 0$. As post-auction profits increase in the own share, there is no gain in demanding less. Suppose bidder j wants to win more than x_j. This is only possible by displacing at least one other winner, so that bidder j's VCG price is at least π^*, which is higher than j's willingness-to-pay. Hence, the profile of bidding functions constitutes a Nash equilibrium. The equilibrium is payoff dominant. Suppose there is an allocation $x' \in Y$ such that $\pi_i(x') \geq \pi_i(x)$ for all bidders and strict for at least one. Then x' was feasible and would maximize bidder welfare, which is clearly a contradiction. □

Proof of Proposition 5. The allocation $\tilde{x}$ maximizes consumer surplus. The VCG auction selects the maximum of π. Hence, when $\tilde{x}$ is a local maximum of π on $\bar{X}$, then sufficiently small caps $\bar{x} > 1/n$ create a ball around $\tilde{x}$ so that the local maximum becomes the global maximum on the restricted domain. The payoff-dominant equilibrium on the restricted domain is then as in the proof of Proposition 4. What is left to show is that $\tilde{x}$ is a local maximum of π in the limit as $s \to \infty$.

First, observe that industry production does not become infinitely large as demand becomes less elastic. This follows from there being a $\bar{Q} > 0$ such that $\tilde{P}(Q) = 0$ for all $Q \geq \bar{Q}$. Firms will never produce an amount such that the market price is zero, so industry output must converge to some number $Q^* < \bar{Q}$. Moreover, because industry production converges, individual production is also finite.

Second, I show that q_i is locally constant in the limit. Equations (2) and (3) provide expressions for $\partial q_1/\partial x_1$ and $\partial q_1/\partial x_2$, respectively. Let $q = q_i(\tilde{x})$. Plugging in $P = s\tilde{P}$ and using the ex post symmetry at $\tilde{x}$, the first derivatives can be written as

$$\frac{\partial q_1(\tilde{x})}{\partial x_1} = \frac{1}{s} \frac{c'(1/n)\left(n\tilde{P}' + (n-1)q\tilde{P}''\right)}{\tilde{P}'\left((n+1)\tilde{P}' + nq\tilde{P}''\right)} \tag{9}$$

$$\frac{\partial q_1(\tilde{x})}{\partial x_2} = \frac{1}{s} \frac{-c'(1/n)\left(\tilde{P}' + q\tilde{P}''\right)}{\tilde{P}'\left((n+1)\tilde{P}' + nq\tilde{P}''\right)}. \tag{10}$$

The second factors on the right-hand sides converge to constants as $s \to \infty$. As a result, the two partial derivatives tend to 0.

Ex ante symmetry and local independence of q_i of x imply that $q_i(x) = Q^*/n$ for all x in a neighborhood of $\tilde{x}$ for s sufficiently large. Industry profits are then locally equal to $P(Q^*)Q^* - Q^*/n \sum c_i(x_i)$ and maximized by minimizing the sum of marginal costs. This is done by allocation $\tilde{x}$. □

Proof of Proposition 6. I first show that $\tilde{x}$ and not $\hat{x}$ maximizes consumer surplus. Let k be the number of active firms and let the supply be split evenly among these firms. Note that only such allocations can maximize consumer surplus when the CRT is strictly convex. In the following, I treat k as a continuous variable with $k \in (1, n]$ and use the implicit function theorem to show that $Q'(k) > 0$. To this end, consider the sum of k first-order conditions $F(Q, k) = kP(Q) + P'(Q)Q - kc(1/k)$. The first derivatives of F are $F_Q = (k+1)P' + P''Q$ and $F_k = P - c(1/k) + c'(1/k)/k$, respectively. The implicit function theorem implies

$$Q'(k) = -\frac{P - c(1/k) + c'(1/k)/k}{(k+1)P' + P''Q}.$$

The denominator is negative due to the decreasing marginal-revenue property. Hence, if $P - c(1/k) + c'(1/k)/k > 0$ then $Q'(k) > 0$. Observe that $P - c(1/k) + c'(1/k)/k \geq P - c(1/k) + \int_0^{1/k} \rho'(t)dt = P - \theta - \rho(1/k) + \rho(1/k) = P - \theta > 0$. The last inequality is true because in the Cournot continuation equilibrium in which a firm wins nothing, its profits are positive, i.e., $(P - c(0))q_i = (P - \theta)q_i > 0$. As aggregate output increases in the number of active firms, consumer surplus is maximized by $\tilde{x}$.

I now show that industry profits as a function of the number of active firms k decrease in k whenever every active firm receives $1/k$ licenses. Industry profits are

$\pi(k) = Q(k)(P(Q(k)) - c(1/k))$. The first derivative is

$$\pi'(k) = Q'(k)(P - c) + Q(P'Q'(k) + c'/k^2) = Q'(k)(P - c + P'Q) + Qc'/k^2.$$

The sum of k first-order conditions requires $P'Q = kc(1/k) - kP$. Plugging in yields

$$\pi'(k) = Q'(k)(k - 1)(c - P) + Qc'/k^2.$$

The first derivative is negative as $Qc'/k^2 < 0$, $Q' > 0$, and $P > c$. As a result, the producer-surplus maximizing VCG auction never selects $\tilde{x}$ because $\pi(\hat{x}) > \pi(\tilde{x})$. □

B Extension: Differentiated Products

Are there differentiated Bertrand and Cournot markets for which firms have preferences over shares and consumers are indifferent between all no-undersell allocations? To address this question, it is useful to recall the key features that lead to these properties in homogeneous Cournot competition. First, the sum of marginal costs is the same for all no-undersell allocations if and only if the cost-reduction technology is linear. Second, Cournot competition is an aggregative game in which the sum of marginal costs determines the aggregate $Q = \sum_i q_i$. Third, consumer surplus only depends on the aggregate, thus on the sum of marginal costs. Fourth, firms' preferences depend on Q_{-i} (or on Q when considering inclusive best replies), which depends on the sum of (other firms') marginal costs. So, to generalize the result, I consider aggregative games in which (1) the sum of marginal costs determines the aggregate and (2) the aggregate determines consumer surplus.

An oligopoly game is aggregative if a firm's payoff depends only on its own action a_i and the aggregate of all firms' actions $A = \sum_{i=1}^{n} a_i$. A firm's best reply is a function of A_{-i} in an aggregative game. The *inclusive best reply* (IBR) is a function of A. Anderson et al. (2020) characterize consumers' indirect utility functions for differentiated Bertrand and Cournot competition so that the aggregate determines consumer surplus. Translated to models of competition with at least three firms, Bergstrom and Varian (1985) show that the sum of marginal costs determines the aggregate if and only if the inclusive best reply is affine in marginal cost c_i for all firms i and the IBRs have the same slope. I combine these results.

B.1 Differentiated Bertrand Competition

In differentiated Bertrand competition, firm i's profit is given by $\pi_i = (p_i - c_i)D_i(p)$, where p is the profile of prices and D_i firm i's demand. A sufficient condition for the game to be aggregative with consumer surplus depending only on the aggregate is that the representative consumer's indirect utility function is additively separable and of the form $\mathbb{V}(p,Y) = \phi(\sum_j v_j(p_j)) + Y$, where Y is income; ϕ is increasing, twice differentiable, and strictly convex in p_j; and v_j is twice differentiable and decreasing in p_j (Anderson et al., 2020, Proposition 1). The game is aggregative with $a_i = v_i(p_i)$. The following proposition takes this functional form as given.

Proposition 8. *Let $n \geq 3$ and consider differentiated Bertrand competition with firm i's profit given by $\pi_i = (p_i - c_i)D_i(p)$. Let $\mathbb{V}(p,Y) = \phi(\sum_j v_j(p_j)) + Y$ be the representative consumer's indirect utility function, where Y is income; ϕ is increasing, twice differentiable, and strictly convex in p_j; and v_j is twice differentiable and decreasing in p_j. The following statements are equivalent:*

1. *Post-auction consumer welfare is the same for all no-undersell allocations.*
2. *Firms have preferences over shares.*
3. *The cost-reduction technology is linear and $v_j(p_j) = \alpha_j - \beta p_j$ for all $j = 1, 2, \ldots, n$, where $\beta > 0$.*

Firm i's demand is $D_i(p) = -\phi'(A)v_i'(p_i)$ by Roy's identity. If $v_i'(p_i) = -\beta$, the first-order condition of profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = \beta\phi'(A) + (p_i - c_i)(-\phi''(A)\beta^2) = 0,$$

which leads to

$$p_i = c_i + \frac{\phi'(A)}{\beta\phi''(A)}.$$

It follows that the IBR is

$$a_i = \alpha_i - \frac{\phi'(A)}{\phi''(A)} - \beta c_i. \tag{11}$$

Note that all firms have the same mark up $p_i - c_i$ irrespective of the license allocation. Moreover, the demand for each firm is $1/n$ of aggregate demand. As an example, if $\phi(A) = \exp(A)$, then $p_i - c_i = 1/\beta$.

Proof. The first-order condition of profit maximization is

$$\frac{\partial \pi(p;x)}{\partial p_i} = D_i(p) + (p_i - c_i)\left(-\phi''(\sum_j v_j(p_j))v_i'(p_i)^2 - \phi'(\sum_j v_j(p_j))v_i''(p_i)\right) = 0.$$

Plugging in D_i, the aggregate $A = \sum_j v_j(p_j)$, and $p_i = v_i^{-1}(a_i)$ gives

$$(v_i^{-1}(a_i) - c_i)\left(-\phi'(A)v_i''(v_i^{-1}(a_i)) - v_i'\left(v_i^{-1}(a_i)\right)^2\phi''(A)\right) - v_i'(v_i^{-1}(a_i))\phi'(A) = 0. \quad (12)$$

I show (1) $\Rightarrow$ (3). Let post-auction consumer welfare be the same for all no-undersell allocations. Consumer welfare is $\phi(A) + Y$, so the aggregate A must be the same for all allocations $x \in \bar{X}$. By the continuity of c_i, there are no-undersell allocations for which the sum of marginal costs is the same. The main result in Bergstrom and Varian (1985) implies that the inclusive best reply must take the form $f_i(A) + g(A)c_i$ for some functions f_i and g in A.

I show that $v_i' = -\beta$ must hold. The IFT and Equation (12) imply

$$\frac{da_i}{dc_i} = -\frac{\phi'(A)v_i''(p_i) + \phi''(A)v_i'(p_i)^2}{\frac{1}{v_i'}\left(-(p_i - c_i)\left(v_i'''\phi'(A) + 2\phi''(A)v_i'v_i''\right) - 2\phi'(A)v_i'' - \phi''(A)(v_i')^2\right)},$$

where I suppress the argument of the derivatives of v_i in the denominator. The IBR being of the form $f_i(A) + g(A)c_i$ implies that $\frac{da_i}{dc_i}$ must be equal to a constant $g(A)$. In particular, it must be the same for all players and independent of p and c_i. This can only be the case if $v_i' = v_j' = -\beta$ for all i.

I now show that the CRT must be linear. If $v_i(p_i) = \alpha_i - \beta p_i$ for all i, then the IBR is given by Equation (11). The sum of the n inclusive best replies is

$$A = \sum_i \alpha_i - \beta \sum_i c_i(x_i) - n\frac{\phi'(A)}{\phi''(A)}.$$

As A is the same for all $x \in \bar{X}$, it must be that the sum of marginal costs is the same for all $x \in \bar{X}$, which can only be true if the CRT is linear.

I show (3) $\Rightarrow$ (2). Let $v_i(p_i) = \alpha_i - \beta p_i$ for all i and the CRT be linear. The sum of marginal costs is the same for all no-undersell allocations. The IBR is as in Equation (11), so the sum of marginal costs determines the equilibrium aggregate A (Bergstrom and Varian, 1985). Hence, for any $x_i \in [0, 1)$, the IBR is the same for all x_{-i} such that $(x_i, x_{-i}) \in \bar{X}$, which proves that firms have preferences over shares.

I show (2) $\Rightarrow$ (1). Let firms have preferences over shares. Observe that

$$\frac{d \max_{p_i} \pi(p_i, A_{-i})}{dA_{-i}} = (p_i - c_i)\frac{\partial D_i(p_i, A_{-i})}{\partial A_{-i}} < 0$$

by the envelope theorem (higher A_{-i} means lower competitors' prices). Hence, if the equilibrium A_{-i} increased, firm i's equilibrium profits would go down. As the firm has preferences over shares, it cannot be that A_{-i} depends on the specific license allocation. As all firms have preferences over shares, the aggregate A cannot depend on the license allocation, so consumer welfare is the same for all $x \in \bar{X}$. □

B.2 Differentiated Cournot Competition

In differentiated quantity competition, the no-externalities result does not significantly extend beyond the homogeneous case: There are no externalities only if the firms' inverse demand functions differ, at most, by constants. The sufficiency is straightforward: Adding constants to Equation (1) does not change the fact that the sum of marginal costs pins down the aggregate (provided that all firms produce positive amounts for any license allocation).

A Cournot differentiated-products oligopoly game is aggregative with consumer surplus only depending on the aggregate if and only if firm i's inverse demand function takes the form

$$P_i = ZbB_i q_i^{b-1} + k_i + bB_i q_i^{b-1} \xi'(\sum_j B_j x_j^b), \tag{13}$$

for all i, where $Z > 0, \xi' > 0$, $\xi'' < 0$, $b < 1$, and $B_i b > 0$ (Anderson et al., 2020). The aggregative action is $a_i = B_i q_i^b$. The homogeneous case features $b = 1$, $k_i = 0$, and $B_i = B_j$ for firms $i, j = 1, 2, \ldots, n$. Note that the differentiated Cournot model does not nest the original model of Section 2 as it assumes that ξ is strictly concave, whereas I assumed the weaker decreasing marginal-revenue property. The proof of the following proposition is omitted as it is analogous to the proof of Proposition 8.

Proposition 9. *Let $n \geq 3$ and consider differentiated Cournot competition with inverse demand functions as in* (13). *The following statements are equivalent:*

1. *Post-auction consumer welfare is the same for all no-undersell allocations.*
2. *Firms have preferences over shares.*
3. *The CRT is linear and $b = 1$, $B_i = B_j$, and $B_i > 0$ for all $i, j = 1, 2, \ldots, n$*

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